

1. Let  $(h, k)$  be the center of  $x^2 + 4x + y^2 + 6y - 3 = 0$ , and let  $r$  be the radius.  
Let  $d$  be the distance between the vertex and the focus of  $y^2 + 4x + 5 = 0$ .  
Let  $m$  be the slope of  $6x + 2y + 1 = 0$ .  
What is  $h + k + r + d + m$ ?
2. Let  $f(x) = 2x^3 + 3x^2 - 8x + 12$ .  
Let  $R$  be the sum of the reciprocals of the roots of  $f$ .  
Let  $S$  be the sum of the squares of the roots of  $f$ .  
What is  $3R + 4S$ ?
3. Suppose functions  $f$  and  $g$  are defined so that  $f(x + y) = f(x)f(y)$  and  $g(xy) = g(x) + g(y)$ , and  $f$  is nowhere zero. What is  $f(g(1))$ ?
4. Let  $A$  be the number of intersection points between  $xy = 1$  and  $x^2 - 2x + y^2 + 2y + 1 = 0$ .  
Let  $B$  be the number of intersection points between  $x^2 + 2x + y^2 = 0$  and  $x^2 - y^2 = 1$ .  
What is  $A + B^2$ ?
5. How many of the following statements are true?
  - I.  $f(x) = x^2 + 3$  is an even function.
  - II.  $f(x) = \log x$  is an even function.
  - III.  $x^2 + 2y^2 = 1$  is a hyperbola.
  - IV.  $2x^2 - 3y + 4 = 0$  has a latus rectum 6 units long.
  - V.  $f(x) = |x|$  is one-to-one.
  - VI.  $(1 - x)$  is a divisor of  $1 - x^9$ .
6. What value of  $B$  will cause the following function to have exactly five zeros?

$$f(x) = \left| \left| |x| - 1 \right| - 3 \right| - B$$

7. Let  $A$  be the 34<sup>th</sup> term in the arithmetic sequence 1, 4, 7, ....  
Let  $B$  be the sum of the first 21 terms of the arithmetic sequence 2, 6, 10, ....  
Let  $C$  be the ninth term in the geometric sequence 1,  $\sqrt{2}$ , 2, ....  
Let  $D$  be the sum of the infinite geometric series  $\frac{2}{3}, \frac{4}{9}, \frac{8}{27}, \dots$   
Find  $A + B + C + D$ .
8. Consider the following function.  
 $\sigma(x)$  is the sum of the positive integer divisors of  $x$ .  
Evaluate

$$\frac{\sigma(196)}{\sigma(4)\sigma(49)} \frac{\sigma(9)\sigma(25)}{\sigma(225)} \sigma(11)$$

9. Use each answer to determine the next.

Let  $A$  be the sum of the roots of  $y = x^2 + 5x - 36$ .

Let  $B$  be the positive solution of  $\log_2(x^2) = A + 1$ .

Let  $C$  be the slope of the line  $x + By = 2$ .

What is  $C$ ?

10. Suppose

$$\begin{aligned}x + y + z &= 2 \\x - z &= 4 \\2x - y + z &= 3\end{aligned}$$

What is  $x^2 + xy - yz - z^2$ ?

11. Let  $A$  be the  $y$ -intercept of the line parallel to  $4x + 3y = 5$  through the point  $(6, -5)$ .

Let  $B$  be the  $x$ -intercept of the line perpendicular to  $3x - 2y = 1$  through the point  $(4, -4)$ .

Let  $C$  be the  $y$ -intercept of the parabola with focus at  $(4, 3)$  and directrix  $y = -1$ .

What is  $(A + B)(C)$ ?

12. Evaluate

$$1 + \log_2(-1 + 2^{1 + \log_2(-1 + 2^{\dots})})$$

13. The eigenvalues of a matrix  $A$  are all the values of  $\lambda$  which satisfy

$$\det(A - \lambda I) = 0$$

where  $I$  is the identity matrix. Find the sum of the eigenvalues of

$$A = \begin{pmatrix} 1 & 3 \\ 3 & 1 \end{pmatrix}.$$

14. Let  $A$  be the solution set of

$$||x + 1| - 1| = 2$$

Let  $B$  be the solution set of

$$|x - 2| + |x + 2| \geq 6$$

How many integers between  $-5$  and  $5$  inclusive are in  $A \cup B$ ?

15. Let  $W$  be the number of ways 5 men and 4 women can be arranged in a line alternating men and women.

Let  $X$  be the number of ways 6 distinct keys can be arranged on a key ring with a clasp.

Let  $Y$  be the number of distinct ways to arrange the letters in MUALPHATHETA

Let  $Z$  be the number of ways to arrange 6 books chosen from a pile of 12 distinct books.

What is  $\frac{WY}{XZ}$ ?