

CALCULUS TEST
SOLUTIONS

TEAM

2007

Developed by:
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1) Derive a formula to express $\sin 5x$ in terms of $\sin x$.

ANSWER: $16\sin^5 x - 20\sin^3 x + 5\sin x$

2) Find $\int_0^2 (4-x^2)^{\frac{3}{2}} dx$

ANSWER: 3π

3) On what interval(s) is the function $f(x) = 3x^5 - 5x^3$ decreasing, concave down?

ANSWER: $\left(-1, -\frac{1}{\sqrt{2}}\right) \cup \left(0, \frac{1}{\sqrt{2}}\right)$

4) A metal plate has a surface temperature distribution given by $T(x, y) = 250 - 4x^2 - 6y^2$. Find the path that a heat-seeking particle would follow if it were placed at the point (2,4)

ANSWER: $y^2 = 2x^3$

5) Find $\int_0^1 \int_{3x}^3 (e^{-y^2}) dy dx$

ANSWER: $\frac{1 - e^{-9}}{6}$

6) Find $15 \int (\tan^6 x) dx$

ANSWER: $3 \tan^5 x - 5 \tan^3 x + 15 \tan x - 15x + C$

7) If $\lim_{x \rightarrow \infty} \sqrt{Ax^2 - 3Bx} - 2Cx = 6$, find $\frac{B^2 - 4C^2}{5A}$

ANSWER: 3

- 8) $f(x) = ax^3 + bx^2 + c$ has a local maximum at $(2,3)$ and a point of inflection at $(1,5)$. Find the value of $f(4)$.

ANSWER: 23

- 9) Use Newton's method with $x_0 = 1$ to solve the equations to 3dp:

A. $x^3 + x - 1 = 0$

B. $x^3 - \cos x = 0$

Find $A + B$

ANSWER: 1.548

- 10) Find the interval common to:

A. The interval of convergence for $\sum_{n=1}^{\infty} \frac{n+1}{n+2} \left(\frac{x}{3}\right)^{n+4}$

B. The interval of convergence for $\sum_{n=1}^{\infty} \frac{(x-2)^{n+1}}{n3^{n+2}}$

ANSWER: $[-1,3)$

- 11) Find $3A - 4B$

A. Area of common interior of $r = 2$ and $r = 4 \cos(4\theta)$

B. Area of common interior of $r = 1 + \sin \theta$ and $r = 3 \sin \theta$

ANSWER: $11\pi - 12\sqrt{3}$

12) Find

A. $2\int\sqrt{1-e^{2x}}dx$ ANS: $2\sqrt{1-e^{2x}} + \ln\left|\frac{\sqrt{1-e^{2x}}-1}{\sqrt{1-e^{2x}}+1}\right| + C$

B. $2\int e^x\sqrt{1-e^{2x}}dx$ ANS: $e^x\sqrt{1-e^{2x}} + \sin^{-1}(e^x) + C$

C. $3\int e^{2x}\sqrt{1-e^{2x}}dx$ ANS: $C - (1-e^{2x})^{\frac{3}{2}}$

13) Plane A contains the points $(1,1,0)$, $(2,-1,5)$ and $(-1,-3,2)$. Plane B contains the points $(3,4,-3)$, $(-1,2,3)$ and $(1,1,2)$. Plane C contains the points $(3,1,-1)$, $(-4,-5,3)$ and $(1,5,-1)$. Find the point lying on planes A, B and C.

ANSWER: $(2,3,-1)$

14) Derive a formula for $\int \sin(mx)\cosh(nx)dx$

ANSWER: $\frac{n \sin(mx)\sinh(nx) - m \cos(mx)\cosh(nx)}{m^2 + n^2} + C$

15) Find $\ln\left[C^{\frac{D}{A}} + 4B\right]$ where

A. $\lim_{x \rightarrow 0^+} \left(\frac{1}{\ln(x+1)} - \frac{1}{x} \right)$

B. $\lim_{x \rightarrow 0^+} (\ln x \cdot \sin x)$

C. $\lim_{x \rightarrow 0^+} (1+3x)^{\frac{1}{2x}}$

D. $\lim_{x \rightarrow 0^+} x^x$

ANSWER: 3