

Question 1

Let $S = \begin{bmatrix} \sin 60^\circ & \cos 60^\circ \\ \cos 60^\circ & \sin 60^\circ \end{bmatrix}$ and $T = \begin{bmatrix} \cos 30^\circ & \sin 30^\circ \\ \sin 30^\circ & \cos 30^\circ \end{bmatrix}$.

Find the determinant of ST .

Question 2

Find the value of $2 + \frac{1}{1 + \frac{1}{2 + \frac{1}{1 + \frac{1}{2 + \frac{1}{\ddots}}}}}$.

Question 3

Let $a = \log_4(e^{3 \ln 4})$

$b = |3i^{741} - 4i^{954}|$

$c =$ sum of x and y which are the solutions of $\begin{cases} 9x + 7y = 2 \\ 4x + 3y = -5 \end{cases}$

Then evaluate $\frac{a^2 - ab}{|a + b - c|}$

Question 4

Carmen has 2007 cans of paint. What is the maximum number of colors she has if she is guaranteed to have at least 345 cans of each color?

Question 5

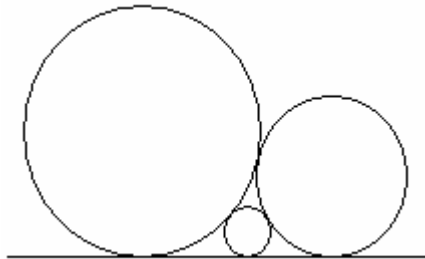
Evaluate: $\lim_{x \rightarrow 2} \frac{\sqrt{3x-5}-1}{x^3-8}$

Question 6

Evaluate: $\cos \left[\frac{\pi}{4} + \frac{\pi}{2} + \frac{3\pi}{4} + \pi + \dots + 22\pi \right]$

Question 7

In the picture below the largest circle has radius of 2 and the middle-sized circle has radius of 1. Find the radius of the smallest circle.

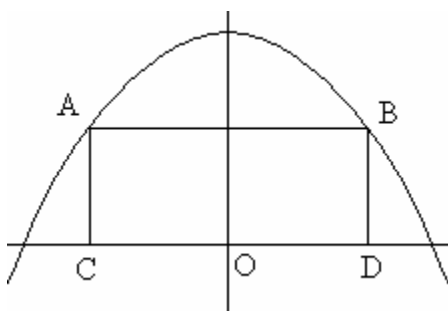


Question 8

For what values of x will the functions $f(x) = \frac{\sqrt[4]{x-2}}{x-5}$ and $g(x) = \ln(3 - \sqrt{x-1})$ both be defined?

Question 9

A rectangle $ABCD$ has side CD on the x -axis, and touches the graph of $y = k \cos(x)$ at points A and B as shown. If the length of CD is $\frac{\pi}{3}$ and area of the rectangle is $\frac{5\pi}{3}$. Find the value of k .



Question 10

Fill in each square below with one of the following symbols $+, -, \div, \times$ in such a way that all the below conditions are satisfied.

- a) Every row must contain all four symbols.
- b) Every column of squares must contain all four symbols.
- c) When all boxes are filled each resulting equation must be valid.

10		1	4		=	10		2	1	
2		1	2		=	4		10	=	
12		2	=		=	3		2	1	
3		11	6		=	3		=	3	
8	=	3	5		=	2		10	2	

Question 11

Let A = the distance between $3x - 4y = 7$ and $3x - 4y = -3$
 Let B = the slope of a line perpendicular to $5x - 2y = 6$

Find $\frac{A}{B}$

Question 12

Find the smallest positive integer in which, when divided by 9 the remainder is 6 and when divided by 17 the remainder is 13.

Question 13

Notice $1 = \frac{1}{2} + \frac{1}{3} + \frac{1}{6}$, can you find 5 different integers

a, b, c, d, e so that $1 = \frac{1}{a} + \frac{1}{b} + \frac{1}{c} + \frac{1}{d} + \frac{1}{e}$. Give 10 different

integers a_1, a_2, \dots, a_{10} so that $1 = \frac{1}{a_1} + \frac{1}{a_2} + \dots + \frac{1}{a_{10}}$.

Question 14

Suppose you have a magic money machine into which you can put in any number of dollar bills. If you insert n dollars, the machine returns $2n$ dollars. Each time you use the machine however, you must insert more money than you did the previous time. If you start with \$1 and use the machine once, you will have \$2. The next time you use the machine you must insert \$2 yielding \$4, and on the third use of the machine you can insert either \$3 or \$4 yielding a total of \$7 or \$8. Notice there is no way that you can obtain exactly \$3 or \$5 or \$6 by using the machine repeatedly starting with \$1. Find the largest amount of money it is impossible to obtain exactly with the magic money machine, starting with \$1.

Question 15

Solve each of the following:

- a) Factor the following polynomial $x^4 + x^2 + 1$
- b) Find all solutions in $[0, 2\pi)$ to the equation $\cos(x) + \sin(2x) = 0$
- c) Let $f(x) = ae^{bx}$, $f(1) = 3$ and $f(3) = 6$. Find a and b .
- d) What is the period of $f(x) = \cos^2(2x)$