

1.

$$P = \lim_{x \rightarrow 0} \frac{1}{2+x} - \frac{1}{2-x}$$

$$Q = \lim_{x \rightarrow 0} \frac{\sin 2x}{x}$$

$$R = \lim_{x \rightarrow \infty} \frac{\ln x^2}{x}$$

$$S = \lim_{x \rightarrow 0} \frac{e^x - 1 - x}{x^2}$$

Find $P + Q + R + S$.

2. If L = the y-intercept of the line tangent to $y = \sin \sin x$ at $\pi, 0$,

$$M = f'(0) \text{ if } f(x) = \frac{\sin^2 x}{\cos x},$$

$$\text{and } N = g'(0) \text{ if } g(x) = \tan \sin x,$$

find $L + M + N$.

$$3. \text{ Suppose } f(x) = \begin{cases} \frac{3x^2 - 3x}{x^2 - 3x + 2} & \text{if } x \neq 1, 2 \\ -3 & \text{if } x = 1 \\ 4 & \text{if } x = 2 \end{cases}.$$

Of the following, which statement(s) are true?

- I. f is continuous for all x II. $\lim_{x \rightarrow 2} f(x)$ exists
III. $\lim_{x \rightarrow 1} f(x)$ exists IV. f is continuous at $x = 1$

4. Find y'' if $\sqrt{x} + \sqrt{y} = 1$. Choose the best answer.

A. $-\frac{\sqrt{y}}{\sqrt{x}}$

B. $\frac{1 - \sqrt{y}}{2x}$

C. $\frac{1}{2x\sqrt{x}}$

D. $\frac{\sqrt{x} - \sqrt{y}}{2x\sqrt{x}}$

5. A 15 m long water trough has cross-sections in the shape of an isosceles trapezoid that is 60 cm wide at the bottom, 100 cm wide at the top, and has height 50 cm. If water is pumped into the trough at a constant rate of $0.5 \text{ m}^3/\text{min}$, how fast is the water level rising when the water is 30 cm deep?

7. Let $F(x) = \int_3^x t^2 - 3t \, dt$,

$$G(x) = \int_x^5 \sqrt{25 - t^2} \, dt,$$

$$H(x) = \int_{-3}^{x^2} e^t - t^3 \, dt, \text{ and}$$

$$J(x) = \int_4^{\cos x} t + \sqrt{1-t} \, dt.$$

Find $F'(4) + G'(3) + H'(1) + J' \pi/2$.

6. Let $g(x) = 5x^{2/3} + x^{5/3}$. If

P = the smallest critical number of $g(x)$,

Q = the x-coordinate of the relative maximum of $g(x)$,

R = the x-coordinate of the point of inflection of $g(x)$,

and S = the y-coordinate of the point of inflection of $g(x)$,

find PQRS.

8. If $A = \int_{-1}^{e-2} \frac{x}{x+2} \, dx$,

$$B = \int_0^1 \frac{\sqrt{\arctan x}}{1+x^2} \, dx,$$

$$C = \int_{-3}^1 \frac{3x}{x^2 + 2x - 8} \, dx, \text{ and}$$

$$D = \int_1^{\infty} \frac{1}{x^2} \, dx,$$

find $A + B + C + D$.

9. Four solids can be formed by revolving the region bounded by the curves $y = x^2 + 1$, $x = 1$, the x-axis, and the y-axis (a) about the x-axis, (b) about the line $y = 2$, (c) about the y-axis, and (d) about the line $x = 1$. What is the sum of the volumes of these four solids?

11.

Find the sum of the 4th degree Maclaurin series representations of $\frac{1}{1-x}$, e^x , $\sin x$, and $\cos x$.

10. Which of the following sequences converge to 0?

I. $a_n = \cos\left(\frac{2}{n}\right)$

II. $a_n = \frac{3n-1!}{3n+1!}$

III. $a_n = \frac{e^n - e^{-n}}{e^{2n} - 1}$

IV. $a_n = n \sin\left(\frac{1}{n}\right)$

V. $a_n = \sqrt{n} - \sqrt{n^2 - 1}$

VI. $a_n = \ln n + 1 - \ln n$

12.

Find the highest point on the curve $x^2 + xy + y^2 = 27$.

13.

Find the sum of the series

$$1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{5} + \frac{1}{8} + \frac{1}{10} + \frac{1}{16} + \frac{1}{20} + \dots$$

where the terms are the reciprocals of the positive integers whose only prime factors are 2's and 5's.

14.

Let $f(x) = \cos x^4$. Find $f^{(16)}(0)$.

1. $\frac{9}{4}$
2. $\pi + 1$
3. III, IV
4. C.
5. $\frac{5}{126}$ m/min (0.040 m/min) **OR** $\frac{250}{63}$ cm/min (3.968 cm/min)
6. 24
7. $2e - 3$
8. $\frac{1}{12}\pi^{3/2} + \ln 5 + e - 2$
9. 8π
10. II, III, VI
11. $3 + 3x + x^2 + x^3 + \frac{13}{12}x^4$
12. (-3,6)
13. $\frac{5}{2}$
14. $\frac{16!}{4!}$ (871,782,912,000)