Calculus Individual Test 2009 Wando Mathematics Meet

For all questions, "E. NOTA" means none of the answers is correct. Unless otherwise stated, assume all numbers are real.

1. Identify the interval(s) below where the function $f(x) = x\sqrt{16 - x^2}$ is decreasing.

a) $-\infty, -\sqrt{8} \cup \sqrt{8}, \infty$ b) $-\sqrt{8}, \sqrt{8}$ c) $-4, -\sqrt{8} \cup \sqrt{8}, 4$ d) $-4, 4$ e) NOTA

2. Identify the interval below where the function $f(x) = 4x^3 - 3x^2 + 5x - 7$ is concave up.

a) $\left(-\infty, -\frac{1}{4}\right)$ b) $\left(-\frac{1}{4}, \infty\right)$ c) $\left(-\infty, \frac{1}{4}\right)$ d) $\left(\frac{1}{4}, \infty\right)$ e) NOTA

3. Find the radius of convergence of the power series $\sum_{n=o}^{\infty} \frac{(-1)^n n\, x^n}{5^n}$

a) $\frac{1}{5}$ b) 5 c) $\frac{e}{5}$ d) $\frac{5}{e}$ e) NOTA

4. A particle moves in the xy-plane so that at any time t, $x = 5t$ and $y = t^2 - 3t$. The particle is at its minimum speed at time t =

a) 0 b) $\frac{1}{2}$ c) $\frac{3}{2}$ d) 2 e) NOTA

5. If $\frac{x+2}{x^3 + 3x^2 - 4x}$ is decomposed into partial fractions, then the coefficient of $\frac{1}{x-1}$ is

a) $\frac{3}{5}$ b) 1 c) -1 d) $-\frac{3}{5}$ e) NOTA

6. Set up the integral that finds the area of the region bounded by the graph of $r = 3 + 2\cos\theta$.

a) $\int_0^{2\pi} 3 + 2\cos\theta \,^2 d\theta$ b) $2\int_0^{2\pi} 3 + 2\cos\theta \,^2 d\theta$ c) $\int_0^{\pi} \frac{1}{2} 3 + 2\cos\theta \,^2 d\theta$

d) $\int_0^{\pi} 3 + 2\cos\theta \,^2 d\theta$ e) NOTA

7. Moving in the xy-plane at time t > 0, the velocity vector of a particle is $4t^3, e^t$. Find its general position vector (in terms of t) for the particle if its position vector at t = 2 is $14, e^2 - 3$.

a) $14, e^2 - 3$ b) $12t^2, e^t$ c) t^4, e^t d) $t^4 - 2, e^t - 3$ e) NOTA

8. If $x = 2t^3 - 4$ and $y = 3t^2$, then $\frac{d^2y}{dx^2} =$

a) $\frac{1}{t}$ b) $\frac{-1}{6t^4}$ c) $\frac{-1}{t^2}$ d) $6t^4$ e) NOTA

9. Let A be the region bounded by $y = e^x$, the y-axis, and the line $y = e$. Which of the following represents the volume of the solid generated when A is revolved around the y-axis?

a) $\pi \int_1^e \ln y^{\,2}\, dy$ b) $\pi \int_0^1 \ln y^{\,2}\, dy$ c) $\pi \int_0^1 e^{2x} dx$ d) $\pi \int_1^e e^{2x} dx$ e) NOTA

10. Find the radius of convergence of $\sum_{n=1}^{\infty} \frac{n^n}{n!} x^n$.

a) $\frac{1}{e}$ b) 1 c) e d) ∞ e) NOTA

11. When the value of sin 3 is approximated by using the 5th degree Maclaurin polynomial for $\sin x$, the value of sin 3 is

a) $1 - \frac{9}{2} + \frac{81}{24}$ b) $-1 + \frac{9}{2} - \frac{81}{24}$ c) $3 - \frac{27}{6} + \frac{243}{120}$ d) $-3 + \frac{27}{6} - \frac{243}{120}$ e) NOTA

12. The slope of the line normal to the curve $f(x) = 3\sin 6x$ at $x = \frac{\pi}{36}$ is

a) $\frac{-\sqrt{3}}{27}$ b) $\frac{-2\sqrt{3}}{9}$ c) $\frac{3\sqrt{3}}{2}$ d) $9\sqrt{3}$ e) NOTA

13. Let $f(x)$ be a differentiable function of x and let $\frac{d(f(x))}{dx} = g(x)$. If $y = (1 + f(x))^3$, then $\frac{dy}{dx} =$

a) $3\ 1 + f(x)^{\,2}$ b) $3\ 1 + f(x)^{\,2} \bullet g(x)$ c) $3\ 1 + g(x)^{\,2} \bullet g'(x)$ d) $3\ 1 + f(x)^{\,2} \bullet f(x)$ e) NOTA

14. If $f(x) = \sin x$, for $\frac{\pi}{2} \le x \le \frac{5\pi}{2}$, then a value for c as prescribed by Rolle's Theorem is

a) π b) $\frac{3\pi}{2}$ c) 2π d) $\frac{7\pi}{2}$ e) NOTA

15. The region bounded by the x-axis and the part of the graph of y = cos x between $x = \frac{-\pi}{2}$ and $x = \frac{\pi}{2}$ is separated into 2 regions by the line x = k. If the area of the region for $\frac{-\pi}{2} \le x \le k$ is 4 times the area of the region for $k \le x \le \frac{\pi}{2}$, then k =

a) $-\frac{\pi}{4}$ b) $\frac{\pi}{4}$ c) $\arcsin\left(\frac{1}{4}\right)$ d) $\arcsin\left(\frac{3}{5}\right)$ e) NOTA

16. If x = 2 is a vertical asymptote of a rational function f, then which of the following must be true?

a) $\lim_{x \to 2} f(x) = \infty$ b) $\lim_{x \to \infty} f(x) = 2$ c) $\lim_{x \to 0} f(x) = 2$ d) $\lim_{x \to 2} f(x) = 0$ e) NOTA

17. If $xy^2 + y^2x = 4$, then $\frac{d^2y}{dx^2}$ at the point (2, 1) is

a) $-\frac{1}{4}$ b) $-\frac{3}{16}$ c) $\frac{3}{16}$ d) $\frac{1}{4}$ e) NOTA

18. The base of a solid is the region in the 1st quadrant bounded by the line $x + y = 4$ and the coordinate axes. What is the volume of the solid if every cross-section perpendicular to the x-axis is a square?

a) 8 b) $\frac{64}{3}$ c) 8π d) $\frac{64\pi}{3}$ e) NOTA

19. Which definite integral below represents the volume of a sphere with radius 3?

a) $\pi \int_{-3}^{3} (x^2 - 9)dx$ b) $\pi \int_{-3}^{3} \sqrt{x^2 - 9}\, dx$ c) $2\pi \int_{0}^{3} (9 - x^2)dx$ d) $2\pi \int_{-3}^{3} \sqrt{4 - x^2}\, dx$ e) NOTA

20. Let $f(x)$ be a continuous and differentiable function on [0, 1], and let $g(x) = f(4x)$. Using the table below, what is the value of $g'(0.1)$?

x	0.1	0.2	0.3	0.4	0.5	0.6
$f'(x)$	2.03	2.045	2.089	2.163	2.302	2.473

a) 2.03 b) 2.163 c) 8.12 d) 8.652 e) NOTA

21. If $f'(x) = 4x^3$ and $f(1) = 4$, then $\int_{0}^{1} f(x)dx =$

a) $\frac{16}{5}$ b) $\frac{21}{5}$ c) $\frac{1}{5}$ d) 1 e) NOTA

22. If 2 equal subdivisions of [0, 4] are used, what is the trapezoidal approximation of $\int_{0}^{4} x^3 - 2x^2 + 7\ dx$?

a) 52 b) 60 c) 90 d) 120 e) NOTA

23. The solution of a differential equation is roughly approximated using Euler's Method and a step of $h = .01$. If $y' = 2x + y$ and $f(2) = 3$, then $f(1.97) = ?$ (to 3 decimal places)

a) 3.213 b) 3.141 c) 2.861 d) 2.793 e) NOTA

24. Find the equation of the tangent line to $y = \tan^{-1}\frac{3x+2}{x-2}$ at x = 1. (Round to the nearest thousandth.)

a) $y + 1.373 = \frac{1}{26}(x - 1)$ b) $y + 1.373 = -\frac{1}{26}\ x - 1$ c) $y + 1.373 = \frac{4}{13}\ x - 1$

d) $y + 1.373 = -\frac{4}{13}\ x - 1$ e) NOTA

25. The linear approximation of $f(x) = \sqrt{\ln x + 2}$ at x = 1 for $f(1.1)$ is (Round to the nearest thousandth.)

a) 1.414 b) 1.445 c) 1.448 d) 1.450 e) NOTA

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Answers

1. C
2. D
3. B
4. C
5. A
6. D
7. D
8. B
9. A
10. A
11. C
12. A
13. B
14. B
15. D
16. E
17. C
18. B
19. C
20. D
21. A
22. B
23. D
24. D
25. D