Wando MAO Math Meet
Spring 2010
Calculus Team Test with Solutions

1. The velocity of a particle as it moves along a number line is given by $v\ t\ = 6t^2 - 4t + 5$. If the position at $t = 1$ is 4, find the sum of the particle's position at $t = 3$ and its acceleration at $t = 5$.

2. L = $f'\left(\frac{\pi}{2}\right)$ if $f(x) = \cos\ \cos x$

M = $g'\ 0$ if $g(x) = \sin\ x \cos x$

Find L – M.

3. Suppose $f(x) = \begin{cases} 2x - 1 \ \text{if}\ x \le 1 \\ 1/x \ \text{if}\ 1 < x < 2 \\ \dfrac{1}{\sqrt{4 - x}} \ \text{if}\ x \ge 2 \end{cases}$. Of the following, which statement(s) are true?

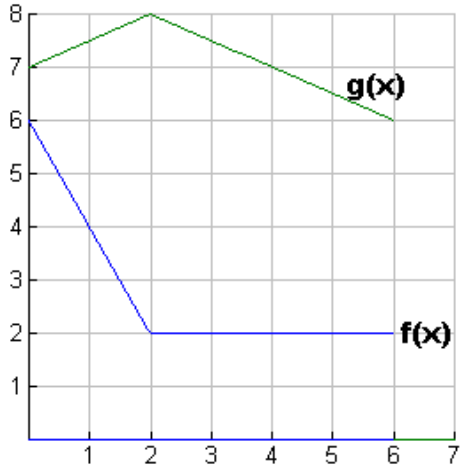
I. f is continuous for all x

II. $\lim_{x \to 2} f(x)$ exists

III. $\lim_{x \to 1} f(x)$ exists

IV. f is continuous at x = 1

4. Let p(x) = g(f(x)) and let r(x) = g(x)f(x). If A = p'(1) and B = r'(4), use the graph below to find A + B.



5. Let $g(x) = x - 4\sqrt{x}$. If

P = the smallest critical number of $g(x)$,

Q = the largest critical number of $g(x)$,

R = the x-coordinate of the minimum of $g(x)$, and

S = the y-coordinate of the minimum of $g(x)$,

find P + Q + R + S.

6. Assign the value of 1 to the true statements below, and assign the value of 0 to the false statements below.

Statement A: If f(x) = $[\![x]\!]$ where $[\![x]\!]$ is the greatest integer $\leq$ x, then $\lim_{x \to -1}$ f(x) does not exist.

Statement B: $\lim_{x \to 3^+} \frac{1}{(x-3)^4} = -\infty$

Statement C: $f(x) = \begin{cases} 18 & x < -3 \\ 2x^2 & -3 \leq x \leq 3 \\ 3x + 9 & x > 3 \end{cases}$ is a continuous function.

What is the value of $\sin^{-1}(A + B + C)$?

7. A = $\lim_{x \to 2} \frac{\sqrt{x+2} - \sqrt{2x}}{x^2 - 2x}$

B = $\lim_{x \to 0} \frac{e^x - 1 - x - x^2/2}{x^3}$

Find A · B.

8. Let $\frac{dy}{dx} = \frac{\ln x}{xy}$.

L = $y(e)$ if $y(1) = 2$ is a point on the graph of y

M = the value of $\frac{d^2y}{dx^2}$ at the point $1,2$

Find $L \cdot M$.

9 A = $\int_0^1 y\ (y^2+1)^5\,dy$

B = $\int_0^{\pi/4} (1+\tan x)^3 \sec^2 x\,dx$

Find A + B.

10. Use the data in the table to find the following:

x	0	2	4	6	8
$f(x)$	10	8	11	14	9

P = the left Riemann sum approximation of $\int_0^8 f(x)\,dx$ with 4 equal subintervals

Q = the right Riemann sum approximation of $\int_0^8 f(x)\,dx$ with 4 equal subintervals

R = the midpoint Riemann sum approximation of $\int_0^8 f(x)\,dx$ with 2 equal subintervals

S = the trapezoidal approximation of $\int_0^8 f(x)\,dx$ with 2 equal subintervals

Find the mean of P, Q, R, and S.

11. Let $F(x) = \int_1^x (1+t^4)^5\,dt$

$G(x) = \int_x^2 \cos(t^2)\,dt$

$H(x) = \int_0^{x^2} \sqrt{1+t^3}\,dt$

Find $F'(1)\cdot G'(\sqrt{\pi})\cdot H'(1)$.

12. A = the minimum value of $f(x) = x^3 - 3x + 1$ on the interval $[0,3]$

B = the rate at which a circle's area is changing (in in^2/sec) when its radius is 3 inches and the radius is changing at a rate of 4 in/sec

Find $A \cdot B$.

Solutions

1. 106
2. -1
3. III, IV
4. 0
5. 4
6. $\pi/2$
7. $-1/48$
8. $\sqrt{5}/2$
9. 9
10. 85
11. $64\sqrt{2}$
12. -24π