Wando MAO Math Meet
Spring 2010
Pre-Calculus Team Test with Solutions

1. Find $A+B+C+D+E$

A: According to the rational root theorem, what is the smallest possible root of $f(x)=-6x^3-x^2+9x-3$

B: The largest possible number of zeros of $g(x)=6x^4+2x^3-x^2+2$

C: The multiplicity of the smallest zero of $h(x)=x(x+2)^3(x-2)^2$

D: State the maximum number of possible turns in a fifth degree equation

E: The x-value of the minimum of $f(x)=-2x^2-x+4$

2. Solve for x

$\log_{27} x+\log_3 x+\log_9 x=11$

3. Solve for y

$$\left(\frac{y+3}{y-1}\right)^2-2=\frac{y+3}{y-1}$$

4. Find $AB+BC+CD$, if

A: The first positive integer solution of $\frac{3x-5}{x+2}\le 2$

B: The sum of the zero(s) of $f(x)=\frac{x^3-4x}{x^4+8x}$

C: The leading coefficient of the oblique asymptote of $f(x)=\frac{4x^4+3x^2-2x+1}{-x^3+x-2}$

D: The value of the abscissa for the removal discontinuity of $f(x)=\frac{2x^2-5x+2}{x^2-4}$

5. Find the sum of the solutions when $\theta \in [0, 2\pi)$.

$$\sin\theta\cos\theta + \frac{1}{2} = 0$$

6. Set A consists of all natural numbers less than 60. A number is selected at random from the set. If it is known that it is prime what is the probability that the number contains a 9?

7. If $\frac{\sin(2\theta)}{2\tan\theta} \bullet \frac{\csc\theta}{\sin\theta} \bullet \frac{\cos\left(\frac{\pi}{2} - \theta\right)}{\sin\left(\theta - \frac{\pi}{2}\right)} = 1$ for $0 < \theta < \pi$

Give the exact value of θ in radians

8. Most people think (hopefully) that $\frac{1}{2} + \frac{1}{3} \neq \frac{2}{5}$. Is it possible that $\frac{1}{a} + \frac{1}{b} = \frac{2}{a+b}$ for some real *a* and *b*? If so, give one ordered pair (*a*,*b*) for which this holds. If not, write "NO" on your answer sheet.

9. $\sqrt{14+\sqrt{180}} = a + b\sqrt{c}$, where *a*, *b*, and *c* are positive integers and *c* is not divisible by the square of any prime.

Find $a+b+c$

10. If any of the parts below have no solution, then assume those parts to have a value of 1337.

Let A be the sum of the real value(s) of *x* which satisfy $9^x - 3^{x+1} = 4$

Let B be the sum of the real value(s) of *x* which satisfy $16^x - 2^{2x+2} = -4$

Let C be the sum of the real value(s) of *x* which satisfy $x^2 + 4x + 6 = 1$

Let D be the sum of the real value(s) of *x* which satisfy $2^{7x+1} = 8^{3x+1}$

What is $3^A + 2B + C + D$?

11. For what value *k* is the following factoring solution valid? (No exponents allowed in answer)

$(2010)^2 + (2010)^3 = k(2011)$

12. Determine, in simplest form, the value of

$$\sin^2\left(\frac{\pi}{8}\right) + \cos^2\left(\frac{3\pi}{8}\right) + \sin^2\left(\frac{5\pi}{8}\right) + \cos^2\left(\frac{7\pi}{8}\right)$$

Solutions

1. $\frac{31}{4}$
2. 729
3. $y = -1, 5$
4. -14
5. $\frac{5\pi}{2}$
6. $\frac{3}{17}$
7. $\frac{3\pi}{4}$
8. NO
9. 9
10. 1341
11. 4,040,100 (a.k.a 2010^2)
12. 2