

# Algebra Review: Linear Equations

Example:  $4x - 2(1 - x) = -38$

$$4x - 2 + 2x = -38$$

$$6x - 2 = -38$$

$$6x = -36$$

$$x = -6$$

Solve each equation:

1)  $2p + 5 = 13$

2)  $12 + 2b = 2 + 5b$

3)  $4x + 5 + 5x + 40 = 180$

4)  $2(4x + 4) = x + 1$

5)  $2(x + 5) = 3(x - 2)$

6)  $180 - x = 3(90 - x)$

7)  $3(180 - y) = 2(90 - y)$

8)  $6x - 3(6 - 5x) + 3x = 10 - 4(2 - x)$

9)  $\frac{1}{2}(6 + 4x) - \frac{1}{4}(8x - 12) = \frac{1}{2}(2x - 4)$

10)  $5x - [7 - (2x - 1)] = 3(x - 5) + 4(x + 3)$

# Algebra Review: Proportions

Definition:  $\frac{a}{b} = \frac{c}{d}$  if and only if  $ad = bc$

Examples:

$$1) \frac{3}{2} = \frac{y}{22}$$

$$3(22) = 2y$$

$$66 = 2y$$

$$33 = y$$

$$2) \frac{x+4}{5} = \frac{x-2}{3}$$

$$3(x+4) = 5(x-2)$$

$$3x + 12 = 5x - 10$$

$$22 = 2x$$

$$11 = x$$

Solve the following proportions using the format in the examples:

$$1) \frac{7}{2} = \frac{y}{3}$$

$$2) \frac{7}{3} = \frac{21}{x}$$

$$3) \frac{25}{15} = \frac{10}{x}$$

$$4) \frac{10}{6x+7} = \frac{6}{2x+9}$$

$$5) \frac{4}{x-3} = \frac{6}{x+3}$$

$$6) \frac{3x-5}{2} = \frac{x-15}{4}$$

$$7) \frac{2-4x}{-6} = \frac{6x-8}{10}$$

$$8) \frac{x+2}{5} = \frac{4}{x+1}$$

$$9) \frac{2}{x-3} = \frac{x-2}{6}$$

# Algebra Review: Systems of Equations

## Substitution Method

Example:	$y = 5 - 2x$	Solution:	Substitute $5 - 2x$ for $y$ in equation 2
	$5x - 6y = 21$		$5x - 6(5 - 2x) = 21$
			$5x - 30 + 12x = 21$
			$17x - 30 = 21$
			$17x = 51$
			$x = 3$

Solve each system of equations by the substitution method.

1)  $y = 2x + 5$   
 $3x - y = 4$

4)  $8x + 3y = 26$   
 $2x = y - 4$

2)  $x = 8 + 3y$   
 $2x - 5y = 8$

5)  $x - 7y = 13$   
 $3x - 5y = 23$

3)  $3x + 2y = 71$   
 $y = 4 + 2x$

6)  $3x + y = 19$   
 $2x - 5y = -10$

## Elimination Method

Example:  $3x + 4y = -10$

$5x - 2y = 18$  X 2

$3x + 4y = -10$

$10x - 4y = 36$

$13x = 26$

$x = 2$

Now substitute 2 for x and

solve for y

$3(2) + 4y = -10$

$4y = -16$

$y = -4$

Example:  $5x - 2y = -19$  X 3

$2x + 3y = 0$  X 2

$15x - 6y = -57$

$4x + 6y = 0$

$19x = -57$

$x = -3$

$2(-3) + 3y = 0$

$3y = 6$

$y = 2$

Solve each system of equations using the elimination method. Use the format shown in the examples.

1)  $3x + 4y = 9$   
 $-3x - 2y = -3$

4)  $4x - 6y = -26$   
 $-2x + 3y = 13$

2)  $5x + 3y = 30$   
 $3x + 3y = 18$

5)  $2x - 8y = 24$   
 $3x + 5y = 2$

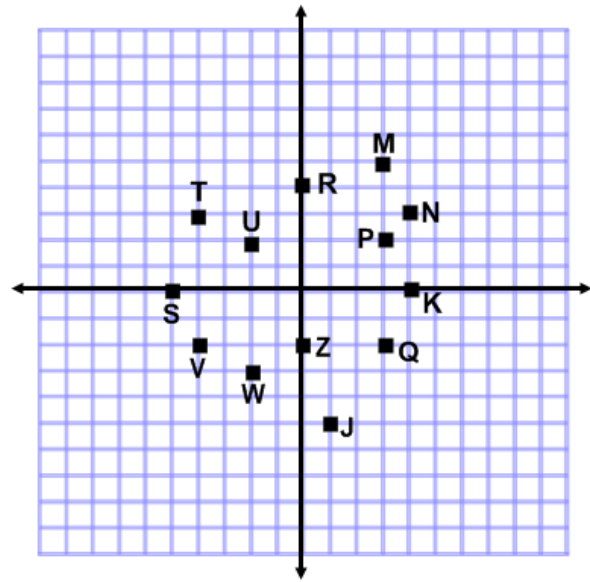
3)  $3x + y = -3$   
 $x + 4y = 10$

6)  $5x - 9y = 47$   
 $6x + 2y = 18$

# Algebra Review: The Coordinate Plane

Name the coordinates of each point:

- |      |       |
|------|-------|
| 1) M | 6) T  |
| 2) N | 7) U  |
| 3) K | 8) V  |
| 4) R | 9) W  |
| 5) S | 10) Q |



11) Name all the points shown that lie on the x-axis.

12) Name all the points shown on the y-axis.

13) What is the x-coordinate of every point that lies on the vertical line through P?

14) Which of the following points lie on a horizontal line through W?

- (-2,1)      (2,3)      (1,-3)      (-2,0)      (0,-3)      (2,0)

Name all the points shown that lie in the quadrant indicated. (A point on an axis is not in any quadrant.)

- 15) Quadrant I      16) Quadrant II      17) Quadrant III      18) Quadrant IV

Plot each point on the graph above.

- |               |              |              |               |
|---------------|--------------|--------------|---------------|
| 19) A (2,1)   | 20) B (5,0)  | 21) C (0,3)  | 22) D (-3,1)  |
| 23) E (-2,-1) | 24) F (1,-2) | 25) G (4,-2) | 26) H (-4,-3) |

## Algebra Review: Fractions

Examples:

$$\text{a) } \frac{8w}{2}$$

$$4w$$

$$\text{b) } \frac{5x-10}{15}$$

$$\frac{5(x-2)}{15}$$

$$\frac{x-2}{3}$$

$$\text{c) } \frac{x+6}{36-x^2}$$

$$\frac{x+6}{(6-x)(6+x)}$$

$$\frac{1}{6-x}$$

$$1) \frac{14}{70}$$

$$2) \frac{75}{15}$$

$$3) \frac{18a}{36}$$

$$4) \frac{3x}{x}$$

$$5) \frac{x}{3x}$$

$$6) \frac{5bc}{10b^2}$$

$$7) \frac{-8y^3}{2y}$$

$$8) \frac{-18r^3t}{12rt}$$

$$9) \frac{3ab^2}{6bc}$$

$$10) \frac{6a+12}{6}$$

$$11) \frac{9x-6y}{3}$$

$$12) \frac{33ab-22b}{11b}$$

$$13) \frac{x+2}{3x+6}$$

$$14) \frac{2c-2d}{2c+2d}$$

$$15) \frac{t^2-1}{t-1}$$

$$16) \frac{5a+5b}{a^2-b^2}$$

$$17) \frac{b^2-25}{b^2-12b+35}$$

$$18) \frac{a^2+8a+16}{a^2-16}$$

$$19) \frac{3x^2-6x-24}{3x^2+2x-8}$$

## Algebra Review: Radical Expressions

Examples:      a)  $\sqrt{56}$                       b)  $\sqrt{\frac{7}{3}}$                       c)  $(3\sqrt{7})^2$

Solutions:      a)  $\sqrt{56} = \sqrt{4 \cdot 14} = 2\sqrt{14}$

b)  $\sqrt{\frac{7}{3}} = \frac{\sqrt{7}}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{\sqrt{21}}{\sqrt{9}} = \frac{\sqrt{21}}{3}$

c)  $(3\sqrt{7})^2 = (3\sqrt{7})(3\sqrt{7}) = 3 \cdot 3 \cdot \sqrt{7} \cdot \sqrt{7} = 9 \cdot \sqrt{49} = 9 \cdot 7 = 63$

Simplifying the following:

1)  $\sqrt{36}$

2)  $\sqrt{81}$

3)  $\sqrt{24}$

4)  $\sqrt{98}$

5)  $\sqrt{300}$

6)  $\sqrt{\frac{1}{4}}$

7)  $\frac{\sqrt{5}}{\sqrt{3}}$

8)  $\sqrt{\frac{80}{25}}$

9)  $\frac{2\sqrt{3}}{\sqrt{12}}$

10)  $\sqrt{\frac{250}{48}}$

11)  $\sqrt{13^2}$

12)  $(\sqrt{17})^2$

13)  $(2\sqrt{3})^2$

14)  $(3\sqrt{8})^2$

15)  $(9\sqrt{2})^2$

16)  $5\sqrt{18}$

17)  $4\sqrt{27}$

18)  $6\sqrt{24}$

19)  $5\sqrt{8}$

20)  $9\sqrt{40}$

Examples:       $2^2 + x^2 = 4^2$

$x^2 + (3\sqrt{2})^2 = 9^2$

$4 + x^2 = 16$

$x^2 + 18 = 81$

$x^2 = 12$

$x^2 = 63$

$x = \pm\sqrt{12}$

$x = \pm\sqrt{63}$

$x = \pm 2\sqrt{3}$

$x = \pm 3\sqrt{7}$

# Algebra Review: Quadratic Equations

Example:  $3x^2 + 14x + 8 = 0$

$$(3x + 2)(x + 4) = 0$$

$$3x + 2 = 0 \text{ or } x + 4 = 0$$

$$x = -2/3 \text{ or } x = -4$$

Solve by factoring:

1)  $x^2 + 5x - 6 = 0$

2)  $x^2 - 7x - 18 = 0$

3)  $x^2 = 20x - 36$

4)  $x^2 + 8x = 20$

5)  $4x^2 + 15 = 17x$

6)  $3x^2 - 13x - 10 = 0$

7)  $6x^2 + 11x - 10 = 0$

8)  $8x^2 + 10x - 25 = 0$

## Algebra Review: Quadratic Equations (Quadratic Formula)

Example:  $x^2 - 5x + 4 = 0$  Solve using the quadratic formula

Quadratic formula:  $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$$x = \frac{-(-5) \pm \sqrt{(-5)^2 - 4(1)(4)}}{2(1)} = \frac{5 \pm \sqrt{25 - 16}}{2} = \frac{5 \pm \sqrt{9}}{2} = \frac{5 \pm 3}{2} = 4, 1$$

Solve each equation by the quadratic formula.

1.  $x^2 + 6x + 9 = 0$

2.  $x^2 - 6x - 40 = 0$

3.  $x^2 + 8x + 15 = 0$

4.  $x^2 - 4x - 12 = 0$

5.  $x^2 + 6x - 4 = 0$

6.  $2x^2 + 6x + 3 = 0$

7.  $2x^2 - 7x - 3 = 0$

8.  $5x^2 - x - 4 = 0$

9.  $5x^2 + x + 3 = 0$

10.  $x^2 - 2x + 3 = 0$

# Algebra Review: Writing equations of lines

**Slope is the ratio of the change in the y-coordinates over the change in the x-coordinates**

$$m = \text{rise/run}$$

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

where  $(x_1, y_1)$  and  $(x_2, y_2)$  are points on the line.

Is the slope of a line the same no matter what two points on the line are given?

Example: Find the slope of the line that passes through  $(3, 6)$  and  $(-4, 4)$ .

1. Substitute into the formula.

$$m = \frac{4 - 6}{-4 - 3}$$

2. Simplify:  $m = -2/-7$  so  $m = 2/7$

Practice: Find the slope of the line that passes through each pair of points.

1.  $(2, 3)$  and  $(-4, 8)$
2.  $(1/4, 1/2)$  and  $(3/4, 3/8)$

**The slope-intercept form of a line is  $y = mx + b$  where  $m$  is the slope and  $b$  is the y-intercept.**

Example: State the slope and y-intercept of  $y = 2x - 5$

Answer: Slope =  $m = 2$       y-intercept =  $b = -5$

Practice: State the slope and y-intercept of the following lines

3.  $y = 4x - 6$

4.  $y = \frac{1}{2}x + 8$

**Write an equation of a line using slope-intercept form or point-slope equation.**

Using slope-intercept form

1. Find the y-intercept or "b". Substitute the slope (m) and the point (x,y) into  $y = mx + b$  and solve for "b".
2. Substitute the slope "m" and "b" into  $y = mx + b$

Example: Write the equation of the line with slope 5 and passing through (-3,-8)

1. Find the y-intercept "b".

Substitute  $m = 5$ ,  $x = -3$  and  $y = -8$  into  $y = mx + b$

$$-8 = 5(-3) + b \text{ Solve for "b"}$$

$$-8 = -15 + b$$

$$7 = b$$

2. Now substitute into the slope-intercept form of a line

$$y = 5x + 7$$

Using Point-Slope equation for a line

$(y - y_1) = m(x - x_1)$  where m is the slope and  $(x_1, y_1)$  is a point on the line

Example: Write an equation in slope-intercept form for the line that has a slope of -5 through (4, -2)

$$(y - (-2)) = -5(x - 4) \quad \text{Substitute}$$

$$(y + 2) = -5x + 20 \quad \text{Simplify}$$

$$y = -5x + 20 - 2 \quad \text{Put in slope-intercept form}$$

$$y = -5x + 18 \quad \text{Simplify}$$

Practice: Write the equation of a line in slope-intercept form for the lines with the following slopes and y-intercepts .

5.  $m = \frac{2}{3}$  through (3, -4)

6.  $m = -4$  through  $(1, -3)$

How do you write an equation of a line given two points?

1. Find the slope using the formula  $m = \frac{y_2 - y_1}{x_2 - x_1}$
2. Now write the equation of the line using point-slope equation or slope-intercept form.

Example: Write the equation of a line through  $(2, -1)$  and  $(3, -3)$

1. Find the slope:  $m = \frac{-3 - (-1)}{3 - 2} \quad m = -2$
2. Substitute into the point-slope equation  
 $(y - y_1) = m(x - x_1)$   
 $y - (-1) = -2(x - 2)$   
 $y + 1 = -2x + 4$   
 $y = -2x + 3$

Practice: Write the equation for the following lines through:

7.  $(3, -6)$  and  $(6, 2)$

8.  $(-7, 2)$  and  $(-3, 5)$